Allais gravity and pendulum effects during solar eclipses explained

T. Van Flandern*

Meta Research, 6327 Western Avenue, NW, Washington, DC 20015-2456

X. S. Yang†

Faculty of Engineering, University of Wales Swansea, Singleton Park, Swansea SA2 8PP, United Kingdom

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Gravitational and other anomalies seen repeatedly in connection with solar eclipses have led to speculation about a possible gravitational shielding effect as the cause. Here we show that an unusual phenomenon that occurs only during solar eclipses, rapid air mass movement for the bulk of the atmosphere above normal cloud levels, appears to be a sufficient explanation for both the magnitude and behavior of the anomaly previously reported in these pages.

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I. INTRODUCTION

A very accurate Foucault-type pendulum slightly increases its period of oscillation and/or changes its plane of swing (by up to 13.5°) at sites experiencing a partial eclipse of the Sun, as compared with any other time. This effect was first noticed by Allais over 40 years ago [1], and both it and related phenomena are now named after him. Some such effect has been seen at several eclipses since then, but also not seen at other eclipses. In recent years, an anomalous eclipse effect on gravimeters has become well-established even under controlled environmental conditions (especially pressure) [2], which some of the pendulum experiments did not have. Several exotic explanations have been proposed, the most interesting of which is a possible shielding effect of the Sun's gravity while the Moon is partly in front of the Sun. The size of the effect as measured with a gravimeter during the 1997 eclipse was roughly \((5 - 7) \times 10^{-9} \, g_{\oplus}\), where \(g_{\oplus}\) is the acceleration of gravity at the Earth's surface (about 1000 cm/s²). So the acceleration of gravity from the Earth seems to decrease during solar eclipses by about \((5 - 7) \times 10^{-6} \, \text{cm/s}^2\) [3,4]. (See Fig. 1.)

However, the upper limit on any gravitational shielding is now set by Lageos satellites, which suffer an anomalous acceleration of only about 3 \(\times 10^{-10} \, \text{cm/s}^2\) during "seasons" where the satellite experiences eclipses of the Sun by the Earth [5]. This seems to rule out the possibility that the eclipse effect on gravimeters might be gravitational in the interesting sense, such as a shielding effect. But interpreting any manifestation of the Allais pendulum effect as gravitational shielding was always problematic because it failed to explain why the effect set in well before the partial stage of an eclipse began and lasted at least half an hour after it ended; why it had two large excursions in local gravity, one near the beginning and one near the end of the partial phase, with little effect near mid-eclipse; and why the effect would act as a decrease in the Earth's gravity instead of an apparent increase. Because the Sun's gravity opposes the Earth's, if the Moon shielded part of the Sun's gravity, this should increase the acceleration of nearby objects toward the Earth by decreasing the component toward the Sun.

II. ANALYTICAL MODEL

As veterans of many eclipse expeditions, we were keenly aware of the sharp temperature drop that accompanies solar eclipses, with onset soon after the partial phase begins. This temperature drop creates air-mass movement into and out of the eclipse zone. This is analogous to what happens for a meteorological front with a large temperature differential on either side in the absence of rapid air circulation to stabilize the two regions of unequal-pressure air. However, in normal meteorological conditions, only the lower atmosphere in and below the cloud decks is strongly affected by large blocks of air with a pressure differential. But over 90\% of the atmosphere's mass is above the cloud levels. This high atmosphere never experiences a comparable sudden temperature drop in a limited geographic region except at times of solar eclipses because it is normally always in continuous, unobscured sunlight by day. Eclipses partially shut off a heat source for the atmosphere above the clouds within the Moon's penumbral (partial eclipse) shadow, and that lost heat creates a relatively steep temperature gradient at altitudes where such a phenomenon normally never exists. Air can then be forced to move in total volumes much larger than are otherwise possible.

Confirming that air-mass movement in fact happens, hourly data recorded on the day of the 1999 eclipse from a site in the partial eclipse zone (north latitude 38.1°, east longitude 20.6°, maximum eclipse magnitude 85%) shows that atmospheric pressure increased sharply by at least 0.6% during the eclipse (from 979 to 985 g/cm²), returning to pre-eclipse levels afterward [6]. (See Fig. 2.) Unfortunately, the hourly data does not have very good time resolution, and is for a location roughly 700 km from the path of totality. But it gives important clues about the magnitude of the air mass movement.

From the kinetic theory of gases, for any given rms speed of air molecules \(\bar{v}\) and air density \(\rho\), pressure \(p\) is given by \(p = \frac{1}{2} \rho \bar{v}^2\). (This is because pressure changes affect all three
dimensions, so the pressure change in one dimension, such as downward, is reduced by a factor of three. If we differentiate this pressure formula, divide by the original formula, and rearrange terms, we get a relation between percentage changes: \( \frac{\delta p}{p} = \frac{\delta \bar{v}}{\bar{v}} \). We do not have measurements of \( \bar{v} \) during the eclipse, but it varies with the square root of absolute temperature, which obviously goes down during the eclipse. If we assume \( \bar{v} \) drops by the same 0.6% change as the pressure rises, this would imply a net change in air density of 1.8%. Other physically reasonable assumptions can lead to percentage changes in air mass ranging from 1.2% to 2.4%; but we will use this median value of 1.8% for our further analysis.

Simple reasoning suggests that the cooler air inside the eclipse zone will decrease in volume (increasing in density) in accord with Boyle’s law as its temperature drops, creating a “low” pressure region with the unusual character that it would extend to great altitudes. This leaves room for warmer air from outside the eclipse zone on all sides of the advancing shadow of the Moon to flow rapidly into the eclipse zone and fill the volume emptied by the cooler, denser air there. This is what happens on a smaller scale across meteorological fronts. When “highs” and “lows” collide, winds are created that attempt to equalize those discordant pressures. Note that for eclipses, the redistribution of air mass would affect broad areas well outside the eclipse zone through this process because those areas are the reservoir from which the extra air mass would be drawn.

If the shadow were static or slowly moving, air would flow deep into the eclipse zone until pressures equalized again for the higher density of air mass present. The greatest density would be found in the center of the shadow where the eclipse is total. However, reality is far from a static situation. The speed of sound is 330 m/s at sea level, and normally changes little with altitude. But the speed of the Moon relative to the Earth averages close to 1000 m/s. The shadow moves at the same speed as the Moon when projected perpendicular to the surface, or faster when projected obliquely. From that speed we must subtract the speed of the observer on the rotating Earth, which is generally in the same direction, but is never faster than 500 m/s at the equator. So on balance, we see that the Moon’s shadow is always moving at supersonic speeds relative to ground observers.

The result is that warmer air from outside the eclipse zone is continually trying to rush toward the cooler regions just inside the shadow, increasing the total mass of air over the ground below. But that air never gets a chance to penetrate very deeply before the shadow has rushed onward, carrying the high-altitude “front” with it faster than air can travel. Hence, the ground barometric pressure is seen to rise during the eclipse, but the amount will be a complex function of the eclipse geometry. Of course, the shadow cools a much larger volume of air than can be above the observer’s horizon. So the production of gravity anomalies at the observer will be dependent on what the upper atmosphere is doing locally as the shadow approaches, covers, and recedes. But we can be
confident that the air mass movement usually makes the gravity anomaly an upward-pointing force relative to normal air (i.e., a weaker downward acceleration of Earth’s gravity) because air drawn from huge volumes well outside the eclipse zone is concentrated into a smaller volume on the periphery of that zone. Correspondingly, the air deficit over a wider region outside the eclipse zone should produce a slight rise in the local gravitational acceleration—an effect suggested by the data in Fig. 1 for about an hour before the first trough and about an hour after the second trough.

From these considerations, we see that air mass flow during eclipses might have the right qualitative behavior to explain the observed Allais gravity anomaly because it occurs mainly near the periphery of the eclipse zone and is in the right direction. The following question now arises: Can an excess air mass of order 1.8% during eclipses be enough to produce a gravitational force of the observed magnitude? This question has not previously come up because the gravimeter was usually protected from any direct effects of temperature and barometric pressure changes. However, we see here that air mass movement toward an eclipse zone can in principle produce a gravitational effect from which the gravimeter cannot be shielded.

To understand the geometry intuitively, we note that the normal net force of the entire atmosphere on an observer is zero because, to good approximation, the atmosphere is a set of spherical shells each of uniform density; and the net force on any point inside any uniform spherical shell is zero. The net force of the air above the observer’s horizon is obviously upward. The net force of the rest of the air around the globe below the observer’s horizon exactly cancels the upward force. Although that air is much farther away, its mass is much greater than the air mass above the observer. So changes in this distant air mass below the observer’s horizon cannot be neglected in calculating the gravitational effects of air on the observer.

In Fig. 3, let \( P \) be any given surface point (e.g., the observer), \( O \) be the origin of the center of the Earth, and \( Q \) be a point in the atmosphere at a height \( z \) above the ground. For the moment, we will approximate the atmosphere as if it were a spherical shell of uniform density with infinitesimal thickness at average height \( z \). In a later step, we will integrate over many such spherical shells with exponentially decreasing density from the ground up. If \( r \) is the radius of the Earth and \( q \) is the distance from point \( P \) to point \( Q \), then the distance from \( O \) to \( Q \) is \( r+z \). We will also need the two angles \( \Omega \cdot O \cdot P = \alpha \) and \( Q \cdot P \cdot O = \beta \). An element of atmosphere with mass \( dm \) at point \( Q \) will exert an acceleration on point \( P \) of magnitude \( da = Gdm/q^2 \), directed toward the mass element, where \( G \) is the universal gravitational constant. We can resolve this into horizontal and vertical components and integrate over all mass elements in the whole atmosphere or any subset of them to get the total acceleration caused by the air mass considered.

First let us examine the forces for normal air symmetrically placed around an observer at point \( P \). By symmetry, the horizontal components (in the \( x \)-coordinate and \( y \)-coordinate directions perpendicular to the \( z \)-coordinate direction) must average to zero. So we can limit our consideration to just the vertical component of the acceleration in the \( z \)-coordinate direction (positive upward from the ground), for which \( da_z = Gdm \cos \beta q^2 \). Then the mass element is \( dm = 2\pi r \rho \sin \alpha dz \), where \( \rho \) is the mean density of air in the shell, \( dz \) is the vertical thickness of the mass element in the \( z \)-coordinate, \( rd \alpha \) is the width of a ring of mass elements of radius \( r \) centered on the \( OP \) axis, and \( 2\pi r \sin \alpha \) is the circumference of the same infinitesimal ring of atmosphere. With this setup, we need only integrate over \( \alpha \) from 0 to \( \pi \) to sum all mass elements at a given height \( z \) and then integrate over all heights \( z \) from 0 to \( \infty \). So the total vertical acceleration \( a_z \) at point \( P \) caused by the atmosphere is

\[
a_z = -2\pi r^2 G \int_0^\infty \rho \left( \int_0^\pi \frac{\sin \alpha \cos \beta}{q^2} d\alpha \right) dz. \tag{1}
\]

Working with the triangle in the figure, we can eliminate \( q \) and \( \beta \) for the independent variable \( \alpha \). The relations we need are

\[
q^2 = (r+z)^2 + r^2 - 2r(r+z) \cos \alpha
\]

\[
= z^2 + 2r(r+z)(1 - \cos \alpha), \tag{2}
\]

\[
\cos \beta = \frac{q^2 - 2rz - z^2}{2q} = \frac{(r+z)(1 - \cos \alpha) - z}{q}, \tag{3}
\]

\[
\sin \beta = \frac{(r+z) \sin \alpha}{q}. \tag{4}
\]

[Equation (4) would be needed for the horizontal force, which we will not develop further here.] To simplify our integral, we will substitute a new independent variable \( \xi = 1 - \cos \alpha \). Then \( d\xi = \sin \alpha d\alpha \), and the range of integration must proceed from \( \xi = 0 \) to \( \xi = 2 \). Making all these substitutions, Eq. (1) becomes

\[
a_z = -2\pi r^2 G \int_0^\infty \rho \left( \int_0^2 \frac{(r+z)\xi - z}{[z^2 + 2r(r+z)\xi]^{3/2}} d\xi \right) dz. \tag{5}
\]
The interior integral here leads to three terms, all proportional to $-1/r^2$. When that constant is factored out, the remaining integral is then a dimensionless quantity $\gamma (0 < \gamma \approx 1)$. The value of $\gamma$ is $+1$ when evaluated at either $\xi = 0$ or $\xi = 2$, showing that the whole atmosphere gives zero net force. $\gamma$ has a value nearer to zero in between. To good approximation, the $z$ dependence of this interior integral can be neglected for just the portion of the atmosphere above or near the observer’s horizon. For that air, the exterior integral becomes simply

$$-\int_0^\infty \gamma \rho_0 e^{-z} - \gamma \rho_0 h,$$

(6)

where $h$ is the scale height of the atmosphere and $\rho_0$ is the density at sea level. This gives the solution we seek, $a_z = 2 \pi G \gamma \rho_0 h$.

For computation, we adopt these numerical values: $G = 6.672 \times 10^{-8} \text{cm}^3 \text{g}^{-2} \text{s}^{-2}$ (universal gravitational constant), $r = 6.37 \times 10^6 \text{cm}$ (radius of the Earth), $\rho_0 = 1.29 \times 10^{-3} \text{g/cm}^3$ (mean sea level density of dry air at standard temperature and pressure), $h = 8.5 \times 10^5 \text{cm}$ (scale height at a typical surface temperature of 16 °C) [8]. $\gamma = +1.0$ (its maximum value). From these, we derive the value $a_z = 4.6 \times 10^{-4} \text{cm/s}^2$, which is the upward-directed gravitational acceleration of the observer due to the atmosphere above his horizon.

III. NUMERICAL MODEL

Now the effect of the atmosphere might easily have been too weak by many orders of magnitude to be of further interest here. But instead we see that this numerical value is already nearly 100 times bigger than the size of the eclipse effect we are seeking, verifying that we are dealing with quantities of a sufficient order of magnitude to produce the gravitational anomaly seen. However, this coefficient is the total upward acceleration for a non-eclipse situation. The standard atmospheric pressure at sea level is 1035 g/cm$^2$. The increase of barometric pressure by 0.6% signifies that more air mass has moved into the eclipse zone, increasing its density by roughly 1.8%. So the upward acceleration might change by a comparably small amount, i.e., about 8.3 $\times 10^{-6} \text{cm/s}^2$). And this is indeed close to the magnitude of the observed changes, $5 \times 10^{-6} \text{cm/s}^2$. We have therefore verified that air mass movement is quantitatively and qualitatively of the correct size and character to explain the gravitational anomalies seen at the times of eclipses.

Unfortunately, computing the details of how air masses move during an eclipse would be a non-trivial meteorological problem, one we will not attempt here. What we can do that is less demanding is to set up a numerical integration of the effect of air mass changes meeting the constraints described above to determine their calculated effect on a gravimeter, for comparison with the observed effect. So we will assume that the air mass flow at subsonic speeds creates a region of higher air mass near the shadow edges with an exponential drop-off to either side, and with the magnitude of this excess air mass scaled approximately by the measured pressure changes in the 1999 eclipse.

The equation to be numerically integrated will be similar to Eq. (1) except that we cannot assume circular symmetry of the atmosphere around the observer. Moreover, we have no information about how air mass flow may change with height above ground, so we will assume that all change occurs at the scale height, about 8.5 km up. With those alterations, the equation we will integrate becomes

$$a_z = -r^2 G h \int_0^2 \frac{(r+z)\xi^2 - z}{(z+2r(z+\xi)^{3/2}} \int_0^{2\pi} \rho(r,\xi) d\eta d\xi.$$

(7)

Two of these quantities need further definition. First, the factor of $2\pi$ in the $\gamma$-coordinate integral shown earlier, $2 \pi r \sin \alpha$, is here replaced with the interior integral in Eq. (7), where $\eta$ is defined in Eq. (8). This integral must be integrated numerically because of the variations in the atmospheric density $\rho$. The $r \sin \alpha$ part of this factor is already incorporated in the exterior coefficient and the interior integral. Second, the density of an element of atmosphere $\rho$ can vary at every point as a function of projected distance of the element from the Moon’s shadow axis, $r_z$. To compute this latter distance, we project the observer, the atmosphere element, and the shadow axis onto a plane passing through the center of the Earth perpendicular to the line from Earth’s center to the observer, and examine the geometry in Fig. 4.

Viewed from space at any given moment, the Moon’s penumbral shadow (wherein observers see at least part of the Sun’s disk obscured) encloses a certain volume of atmosphere which then cools, compresses, and draws more air in from the surrounding regions. The radius of this conical penumbral shadow near the Earth’s surface is typically about $3.5 \times 10^8 \text{cm}$. The shadow usually encompasses an even larger area on the ground because the Earth’s surface is normally inclined at some arbitrary angle $\theta$ (up to 90°) to the shadow cone, making the projected shadow elliptical in shape. So for any given projected distance of the observer from the shadow axis, $X$ (which is the distance PS in the figure), we can calculate the $x,y$ coordinates of the distance SQ. This would immediately be the distance $r_z$ we seek if the projected shadow were circular. But it is elliptical and
elongated along the line to the observer SP if the observer is in the path or totality. So given \( r, X, \alpha, \eta, \theta \), the relations we need are these:

\[
\begin{align*}
x_Q - x_P &= r \sin \alpha \cos \eta, \\
x_S - x_P &= X, \\
r_s &= \sqrt{(x_Q - x_S)^2 \cos^2 \theta + y_Q^2},
\end{align*}
\]  
(8)

We now assume our air mass model, which will consist of a ring of denser air near the edge of the penumbral shadow (radius \( r_p \)), with a normal curve distribution having amplitude \( k \) and half-width \( r_h \). For example, a typical model would be of this form:

\[
\rho = \rho_0 (1 + ke^{-(r_p-r)^2/r_h^2}).
\]  
(9)

We then adopt two sets of values for \( k \) and \( r_h \) that give the best fits to the data in Fig. 1, one set for before mid-eclipse and the other for after. [The two \( k \) values differ because \( \theta \) changes during the three hours between first and last contacts, which slows the progress of the eclipse near the latter, allowing more time for air mass movement. The \( k \) value adopted were 0.013 and 0.018, while \( r_p \approx 900 \text{ km} \) works well for both contacts.] The results of the two numerical integrations with those parameter values are presented together in Fig. 5. This shows that plausible air mass movement models can reproduce the observations, especially since the larger value from this fit to the 1997 gravimeter data (0.018) is the same as that inferred from measured pressure changes in the 1999 eclipse (1.8%—see discussion of Fig. 2). Only future data collection can assess whether or not the real atmosphere behaves as these models do.

**IV. CONCLUSION**

We can therefore conclude that air mass movement during solar eclipses is a significant effect that cannot be neglected when trying to explain the Allais gravity anomaly, and might well be the entire explanation of the effect. However, the gravitational anomaly discussed here is only a few parts per billion of Earth’s own gravitational force, so it is about a factor of 100 000 too small to explain the Allais excess pendulum precession or the change in pendulum swing period (an increase of 1/3000) that sometimes shows up during eclipses. But the relatively sharp changes in barometric pressure during an eclipse can certainly create local air mass movement at ground level, for example, into or out of a building. So experiments that were shielded only from temperature changes but not pressure changes may have experienced an extra and unexpected driving force from local air movement perhaps responsible for these changes, whereas other experiments with better controls would not have experienced them. This is also consistent with the pendulum effect showing up most often in the early experiments, but having no unambiguous detections within the past 30 years when consciousness of the importance of controls was more widespread.

The explanations presented here for the Allais gravity and pendulum anomalies can be tested and further refined. For example, we would learn more about air mass movement unique to eclipses by sending balloons to high altitude and measuring actual changes in wind speeds and air movement at different locations and altitudes above sea level. This will permit more realistic air movement modeling. In the meantime, the much smaller anomalous acceleration seen in data from the first two Lageos satellites still looks like a good candidate for a possibly real gravitational shielding effect [9–11]. The Allais gravity and pendulum effects, however, are clearly not associated with gravitational shielding.

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